AP Precalculus Name_____ Date_____ Per _____

(35 pts) PRACTICE Quiz: FRQ #3, Graphs of the Other Four Trig Functions, and Inverse Trig

1. (2 pts) All points of inflection on the graph of $y = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$ are found on the line $y = p$. Find p.

   (2 pts) Find the period of $f(x) = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$

   (2 pts) Find the range of $f(x) = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$

   (3 pts) Circle all of the following x-values at which the graph of $y = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$ has vertical asymptotes.

   -9 -3 0

   2 3 6

2. (2 pts) Find the period of $f(x) = 4\csc\left(\frac{3}{2}x\right)-1$

   (2 pts) Find the range of $f(x) = 4\csc\left(\frac{3}{2}x\right)-1$

   (3 pts) Find the equations of three vertical asymptotes of the graph of $y = 4\csc\left(\frac{3}{2}x\right)-1$

3. (3 pts) Find all solutions in the interval $0 \le \theta < 2\pi$. Write your answers in terms of arcsin.

$$4\sin\theta + 3 = 2$$

4. (3 pts) Let $p(x) = 12\arcsin(4x - 1)$.
Find all input values in the domain of p that yield an output value of 2π.

5. (3 pts) Find all solutions in the interval $0 \leq \theta < 2\pi$. Write your answers in terms of arctan.

$$\sec^2\theta - \tan\theta - 3 = 0$$

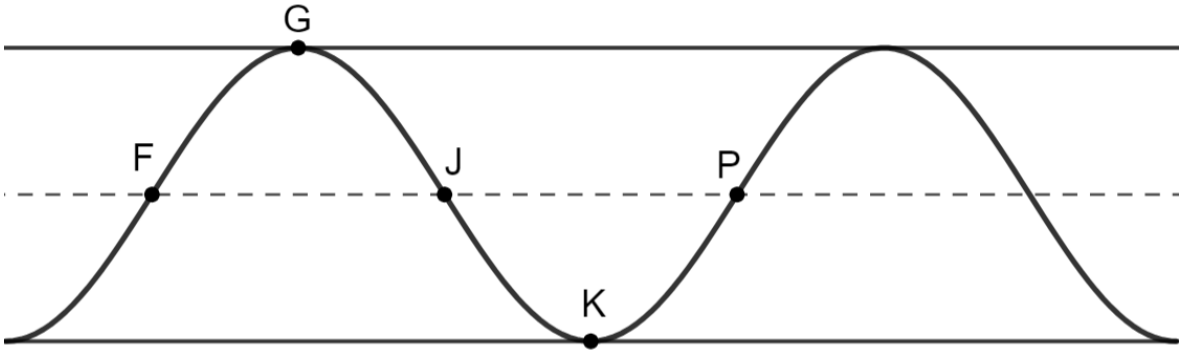
6. (2 pts) Evaluate $\arctan(-1)$

7. (2 pts) Evaluate $\arcsin\left(\frac{1}{2}\right)$

8. The human heart pumps blood around the body. It accomplishes this by periodically contracting and relaxing. At full volume, Christiaan's heart contains $180\ \text{mL}$. After pushing blood out, Christiaan's heart contains $80\ \text{mL}$ of blood. Christiaan's heart beats 50 times per minute, meaning that 50 times each minute, Christiaan goes through the cycle of having his heart full, then roughly half full, then back to full again.

   The sinusoidal function h models the volume of Christiaan's heart, in milliliters, as a function of time t in minutes. Let $t = 0$ refer to a point at which Christiaan's heart is at full volume.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $\big(t, h(t)\big)$ for the five points: F, G, J, K, and P.



F: (,)	G: (,)	J: (,)
K: (,)	P: (,)	

   (B) The function h can be written in the form $h(t) = a\sin\big(b(t + c)\big) + d$. Find values of constants a, b, c, and d.

$a =$	$b =$
$c =$	$d =$

   (C) Refer to the graph of h in part (A). The t-coordinate of F is t_1 and the t-coordinate of G is t_2.
   (i) On the interval (t_1, t_2), which of the following is true about h? (Circle your answer)
      a. h is positive and increasing
      b. h is positive and decreasing
      c. h is negative and increasing
      d. h is negative and decreasing
   (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

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(35 pts) PRACTICE Quiz: FRQ #3, Graphs of the Other Four Trig Functions, and Inverse Trig

1. (2 pts) All points of inflection on the graph of $y = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$ are found on the line $y = p$. Find p.

All POIs on $y=+3$ $p = 3$

(2 pts) Find the period of $f(x) = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$

$b = \frac{\pi}{\text{Period}}$ $\frac{\pi}{4} = \frac{\pi}{\text{Period}}$ Period $=4$ Period $= \frac{\pi}{b} = \frac{\pi}{\pi/4} = 4$

(2 pts) Find the range of $f(x) = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$

Trick Question $(-\infty, \infty)$

(3 pts) Circle all of the following x-values at which the graph of $y = 2\tan\left(\frac{\pi}{4}(x-1)\right)+3$ has vertical asymptotes.

$\frac{\pi}{4}(x-1) = \frac{\pi}{2} + \pi k$

$\frac{1}{4}(x-1) = \frac{1}{2} + k$

$x-1 = 2 + 4k$

$x = 3 + 4k$

−9 −5 −1 3 7 11

−3 2 3 6

2. (2 pts) Find the period of $f(x) = 4\csc\left(\frac{3}{2}x\right)-1$

Period $= \frac{2\pi}{b} = \frac{2\pi}{3/2} = \frac{4\pi}{3}$

(2 pts) Find the range of $f(x) = 4\csc\left(\frac{3}{2}x\right)-1$

$(-\infty, -5] \cup [3, \infty)$

3
-1
-5

(3 pts) Find the equations of three vertical asymptotes of the graph of $y = 4\csc\left(\frac{3}{2}x\right)-1$

$\frac{3}{2}x = \pi k$ $x = \frac{2\pi k}{3}$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{6\pi}{3}, \frac{8\pi}{3}, \ldots$

3. (3 pts) Find all solutions in the interval $0 \le \theta < 2\pi$. Write your answers in terms of arcsin.

$$4\sin\theta + 3 = 2$$

$4\sin\theta = -1$

$\sin\theta = -\frac{1}{4}$

$\alpha = \arcsin\left(\frac{1}{4}\right)$

$\theta = \pi + \arcsin\left(\frac{1}{4}\right)$ $\theta = 2\pi - \arcsin\left(\frac{1}{4}\right)$

4. (3 pts) Let $p(x) = 12\arcsin(4x-1)$.
Find all input values in the domain of p that yield an output value of 2π.

$$2\pi = 12\arcsin(4x-1)$$

$$\frac{\pi}{6} = \arcsin(4x-1)$$

$$\sin\left(\frac{\pi}{6}\right) = \sin(\arcsin(4x-1))$$

$$\frac{1}{2} = 4x-1$$

$$\frac{3}{2} = 4x$$

$$x = \frac{3}{8}$$

5. (3 pts) Find all solutions in the interval $0 \le \theta < 2\pi$. Write your answers in terms of arctan.

$$\sec^2\theta - \tan\theta - 3 = 0$$

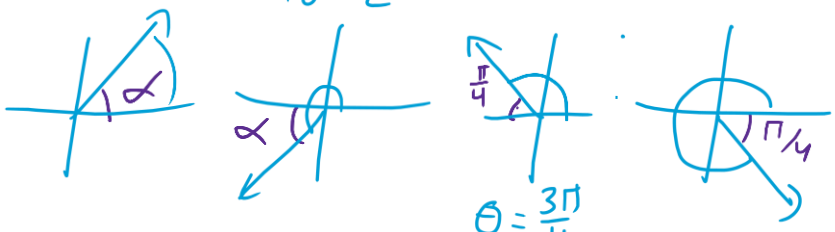
$$\tan^2\theta + 1 - \tan\theta - 3 = 0$$

$$\tan^2\theta - \tan\theta - 2 = 0$$

$$(\tan\theta - 2)(\tan\theta + 1) = 0$$

$\alpha = \arctan(2)$

$\tan\theta = 2$ $\quad$ $\tan\theta = -1$



$\theta = \arctan(2)$ $\quad$ $\theta = \pi + \arctan(2)$ $\quad$ $\theta = \frac{3\pi}{4}$ $\quad$ $\theta = \frac{7\pi}{4}$

6. (2 pts) Evaluate $\arctan(-1)$ $= -\frac{\pi}{4}$

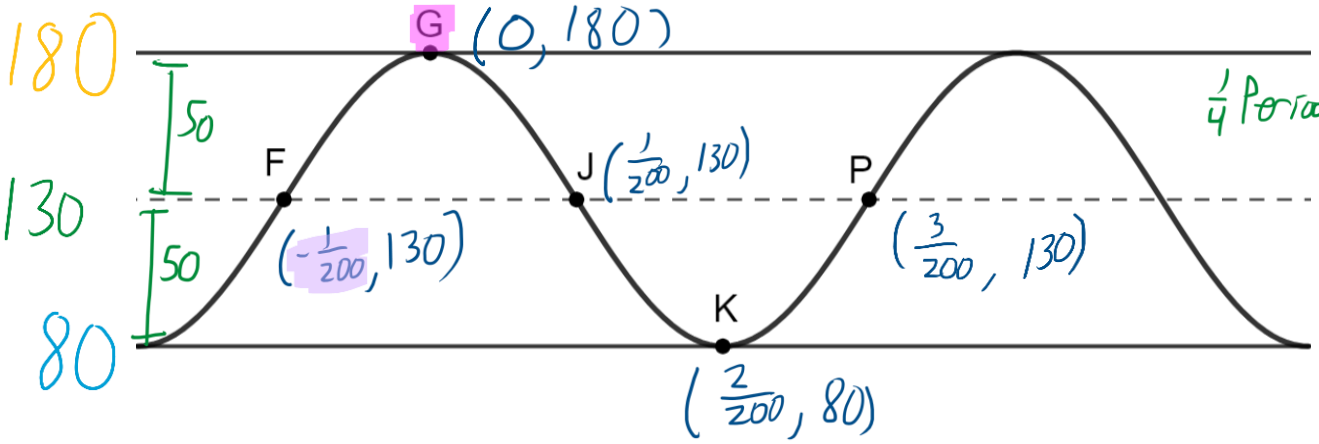
7. (2 pts) Evaluate $\arcsin\left(\frac{1}{2}\right)$ $= \frac{\pi}{6}$

8. The human heart pumps blood around the body. It accomplishes this by periodically contracting and relaxing. At full volume, Christiaan's heart contains 180 mL. After pushing blood out, Christiaan's heart contains 80 mL of blood. Christiaan's heart beats 50 times per minute, meaning that 50 times each minute, Christiaan goes through the cycle of having his heart full, then roughly half full, then back to full again.

The sinusoidal function h models the volume of Christiaan's heart, in milliliters, as a function of time t in minutes. Let $t = 0$ refer to a point at which Christiaan's heart is at full volume.

$\frac{50 \text{ cycle}}{1 \text{ min}}$ $\quad \frac{1}{50} \frac{\text{min}}{\text{cycle}}$ ↓ Period

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



$\frac{1}{4}$ Period $= \frac{1}{4} \cdot \frac{1}{50} = \frac{1}{200}$

F: $\left(-\frac{1}{200}, 130\right)$	G: $(0, 180)$	J: $\left(\frac{1}{200}, 130\right)$
K: $\left(\frac{2}{200}, 80\right)$		P: $\left(\frac{3}{200}, 130\right)$

(B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

$b = \frac{2\pi}{\text{Period}} = \frac{2\pi}{1/50} = 2\pi \cdot \frac{50}{1} = 100\pi$

amplitude

$a = 50$	$b = 100\pi$
$c = \frac{1}{200}$	$d = 130$ ↑ midline

(C) Refer to the graph of h in part (A). The t-coordinate of F is t_1 and the t-coordinate of G is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h? (Circle your answer)

a. h is positive and increasing
b. h is positive and decreasing
c. h is negative and increasing
d. h is negative and decreasing

(ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

On the interval (t_1, t_2), the graph is concave down.
So, the rate of change is decreasing.